

# The effect of different regulators in the non-local field-antifield quantization\*

Everton M. C. Abreu<sup>†</sup>

*Instituto de Física, Universidade Federal do Rio de Janeiro,  
Caixa Postal 68528, 21945-970 Rio de Janeiro, RJ, Brazil  
(February 1, 2008)*

Recently it was shown how to regularize the Batalin-Vilkovisky (BV) field-antifield formalism of quantization of gauge theories with the non-local regularization (NLR) method. The objective of this work is to make an analysis of the behaviour of this NLR formalism, connected to the BV framework, using two different regulators: a simple second order differential regulator and a Fujikawa-like regulator. This analysis has been made in the light of the well known fact that different regulators can generate different expressions for anomalies that are related by a local counterterm, or that are equivalent after a reparametrization. This has been done by computing precisely the anomaly of the chiral Schwinger model.

03.70.+k, 11.10.Ef, 11.15.-q

## I. INTRODUCTION

The non-local regularization (NLR) [1–3] gives a consistent way to compute one-loop anomalies of theories with an action that can be decomposed into a kinetic and an interacting part. It can be proved that anomalies at higher order levels of  $\hbar$  can be precisely obtained with this regularization. The main ideas were based on the Schwinger's proper time method [4]. The NLR arranges the original divergent loop integrals in a sum over loop contribution in such a way that the loops, now composed of a set of auxiliary fields, contain the original singularities. To regularize the original theory one has to eliminate these auxiliary fields by putting them on shell. In this way the theory is free of the quantum fluctuations. The preliminary results [5,6] were very well received.

The method developed by Batalin and Vilkovisky (BV method) [7] showed itself to be a very powerful way to quantize the most difficult gauge field theories. For a review see [8–10].

The BV, or field-antifield formalism, provides at the lagrangian level, a general framework for the covariant

path integral quantization of gauge theories. This formalism uses very interesting mathematical objects such as a Poisson-like bracket (the antibracket), canonical transformations, ghosts and antighosts for the BRST transformations, etc. The most important object of this method at the classical level is an equation called classical master equation (CME).

The fundamental idea of this formalism is the BRST invariance. All the fields  $\Phi^A$ , i.e., the set of the classical fields of the theory together with the ghosts and the auxiliary fields, have their canonically conjugated fields, the antifields  $\Phi_A^*$ . With all these elements we construct the so called BV action. At the classical level, the BV action becomes the classical action when all the antifields are equal to zero.

There are two ways to get a gauge-fixed action: by a canonical transformation, and now we can say that the action is in a gauge-fixed basis; or with a choice of a gauge fermion and making the antifields to be equal to the functional derivative of this fermion.

The method can be applied to gauge theories which have an open algebra (when the algebra of the gauge transformations closes only on shell); to closed algebras; to gauge theories that have structure functions rather than constants (soft algebras); and to the case where the gauge transformations may or may not be independent, i.e., reducible or irreducible algebras respectively.

Zinn-Justin introduced the concept of sources of the BRST-transformations [11]. These sources are the antifields in the BV formalism. It was shown also that the geometry of the antifields have a natural origin [12].

At the quantum level, the field-antifield formalism also works at higher order loop anomalies [13,14]. At one loop, with the addition of extra degrees of freedom, causing an extension of the original configuration space, we have a solution for a quantum master equation (QME) that has been obtained as a part that does not depend on the antifields in the anomaly. In general this solution needs a regularization as we will see below. When the Wess-Zumino terms (which cancel the anomaly) can not be found, the theory can be said to have a genuine anomaly. Recently, a method was developed to handle with global anomalies [15].

However, as has been explained above, the solution of the QME is not easily obtained because there is a  $\delta(0)$ -like divergence when the  $\Delta$  operator, a second order differential operator that will be defined in the next section, is applied on local functionals. The details can be seen in ref. [8–10]. Therefore, a regularization method

\*Present address: Departamento de Física e Química, UNESP, Campus de Guaratinguetá

<sup>†</sup>Financially supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP). E-mail: everton@feg.unesp.br

has to be used to cut the divergence in the QME. One of these prescriptions is the Pauli-Villars (PV) regularization method [16–18], where new fields, the PV fields, and an arbitrary mass matrix are introduced. But this method is very useful only at one-loop level. At higher orders, the PV method is still mysterious. Very recently, a BPHZ renormalization [19] of the BV formalism was formulated [20,21]. The dimensional regularization method at the quantum aspect of the field-antifield quantization has been studied in ref. [22]. Finally, an extension of the NLR method to the BV framework has been recently formulated by J. París [23]. The consistency conditions for higher orders anomalies have been studied in the reference [24].

The objective of this paper is to make a comparison between two ways of regularizing the BV formalism using the pure extended NLR. To do this we have analyzed the results of the NLR using two different kinds of regulators.

It is well known that, for some models, the value of the anomaly can depend on the regulator operator that is being used, i. e., we can obtain different but cohomologically equivalent expressions for the anomaly for these models [27]. The chiral Schwinger model (CSM), anomalous at one-loop only, is one of these models which the guise of the expression for the anomaly is dependent of the form of the regulator as has been demonstrated in [28] through the analysis of the Wess-Zumino (WZ) term, which is responsible for the cancellation of the anomaly. It has been used two different regulators and the results have showed two different WZ terms, but the expressions are equivalent after a reparametrization. In this paper we have regularized the CSM within the context of this extended non-local BV regularization calculating the CSM's anomaly. Firstly we have computed the functional traces using a simple second order differential regulator. After this, the Fujikawa-like regulator was adjusted to this modified BV formalism. We show in a precise way that, using these different regulators we can obtain directly the same result for the anomaly.

This work is organized as follows: in section 2 a brief review of the field-antifield formalism has been made. In section 3 the original NLR was depicted. The extended non-local regularization was described in section 4. The computation of the CSM anomaly at one-loop with the two regulators has been calculated in section 5. In the last section, we have summarized the conclusions and final remarks.

## II. THE FIELD-ANTIFIELD FORMALISM

Let us construct the complete set of fields, including in this set the classical fields, the ghosts for all gauge symmetries and the auxiliary fields. The complete set will be denoted by  $\Phi^A$ . Now, one will extend this space with the same number of fields, but at this time, defining the antifields  $\Phi_A^*$ , which are the canonical conjugated vari-

ables with respect to the antibracket structure. This last object is constructed like

$$(X, Y) = \frac{\delta_r X}{\delta \phi} \frac{\delta_l Y}{\delta \phi^*} - (X \longleftrightarrow Y) , \quad (1)$$

where the indices  $r$  and  $l$  denote right and left functional derivatives respectively.

By means of the antibrackets, one can write the canonical conjugation relations

$$(\Phi^A, \Phi_B^*) = \delta_B^A , \quad (\Phi^A, \Phi^B) = (\Phi_A^*, \Phi_B^*) = 0 . \quad (2)$$

The antifields  $\Phi_A^*$  have opposite statistics to their conjugated fields  $\Phi^A$ . The antibracket is a fermionic operation so that the statistics of the antibracket  $(X, Y)$  is opposite to that of the simple product  $XY$ . The antibracket also satisfies some graded Jacobi relations:

$$(X, (Y, Z)) + (-)^{\epsilon_X \epsilon_Y + \epsilon_X \epsilon_Z} (Y, (X, Z)) = ((X, Y), Z) . \quad (3)$$

where  $\epsilon_X$  is the statistics of  $X$ , i.e.  $\epsilon(X) = \epsilon_X$ .

We define a quantity, named ghost number, to the fields and to the antifields. These are integers such that

$$gh(\Phi^*) = -1 - gh(\Phi) . \quad (4)$$

One can then construct an extended action of ghost number equal to zero, the so called BV action, also called classical proper solution,

$$S(\Phi, \Phi^*) = S_{cl}(\Phi) + \Phi_A^* R^A(\Phi) + \frac{1}{2} \Phi_A^* \Phi_B^* R^{BA}(\Phi) + \dots \\ + \frac{1}{n!} \Phi_{A_1}^* \dots \Phi_{A_n}^* R^{A_1 \dots A_n} + \dots , \quad (5)$$

so that it has to satisfy the classical master equation,

$$(S, S) = 0 . \quad (6)$$

This equation contains the complete algebra of the theory, the gauge invariances of the classical action (where  $S_{cl} = S_{BV}(\Phi^A, \Phi_A^* = 0)$ ), Jacobi identities, ...

Gauge fixing is obtained either by a canonical transformation or by choosing a fermion  $\Psi$  and writing

$$\Phi_A^* = \frac{\delta_r \Psi}{\delta \Phi^A} . \quad (7)$$

To obey the ghost number conservation rule in this expression one have to introduce the BRST antighost in the gauge fixing fermion.

At the quantum level the action can be defined by

$$W = S + \sum_{p=1}^{\infty} \hbar^p M_p , \quad (8)$$

where the  $M_p$  are the corrections (the Wess-Zumino terms) to the quantum action. The expansion (8) is not

the only one, but it is the usual one. An expansion in  $\sqrt{\hbar}$  can be made, for example [30]. This will originate the so called background charges, that are useful in the conformal field theory [31].

The quantization of the theory is obtained with the generating functional of the Green functions:

$$Z(J, \Phi^*) = \int \mathcal{D}\Phi \exp \frac{i}{\hbar} [W(\Phi, \Phi^*) + J^A \Phi_A^*] . \quad (9)$$

But the definition of a path integral properly lacks on a regularization framework, as we have observed already, which can be seen as a way to define the measure of the integral. Anomalies represent the non conservation of the classical symmetries at the quantum level.

For a theory to be free of anomalies, the quantum action  $W$  has to be a solution of the QME,

$$(W, W) = 2i\hbar \Delta W , \quad (10)$$

where

$$\Delta \equiv (-1)^{A+1} \frac{\partial_r}{\partial \Phi^A} \frac{\partial_r}{\partial \Phi_A^*} . \quad (11)$$

In the equation (10) one can see that when it is not possible to find a solution to the QME, we have an anomaly that can be defined by:

$$\mathcal{A} \equiv \left[ \Delta W + \frac{i}{2\hbar} (W, W) \right] (\Phi, \Phi^*) . \quad (12)$$

The anomaly can be represented by a  $\hbar$  expansion,

$$\mathcal{A} = \sum_{p=1}^{\infty} \hbar^{p-1} M_p . \quad (13)$$

Substituting (8) in (12) and using (13) one have the form of the  $p$ -loop BRST anomalies:

$$\mathcal{A}_0 = \frac{1}{2} (S, S) \equiv 0 , \quad (14)$$

$$\mathcal{A}_1 = \Delta S + i(M_1, S) , \quad (15)$$

$$\begin{aligned} \mathcal{A}_p &= \Delta M_{p-1} + \frac{i}{2} \sum_{q=1}^{p-1} (M_q, M_{p-q}) \\ &+ i(M_p, S) , \quad p \geq 2 . \end{aligned} \quad (16)$$

The first equation is the known CME. The second one is an equation using  $M_1$ . If, substituting (8) in (10), there is not a solution for  $M_1$  then  $\mathcal{A}$  is called a genuine anomaly.

The anomaly is not uniquely determined since  $M_1$  is arbitrary. The anomaly satisfy the Wess-Zumino consistency condition [29]:

$$(\mathcal{A}, S) = 0 . \quad (17)$$

It was extensively analyzed in ref. [27] that two different regulators furnish consistent anomalies that are related by a local counterterm,

$$i\Delta^{(2)}S = i\Delta^{(1)}S + (S, M_1), \quad (18)$$

where  $M_1$  is a local counterterm.

We will show that we can obtain directly the same result for the anomaly of the CSM using the NLR method with two different regulators.

### III. THE NON-LOCAL REGULARIZATION

As we have stressed in the introduction, the non-local regularization can be applied only to theories which have a perturbative expansion, i.e. for actions that can be decomposed into a free and an interacting part. For much more details, including the diagrammatic part, the interested reader can see the references [1–3, 23, 24]<sup>1</sup>. Here we have explained the main parts of the method.

Let us define an action  $S(\Phi)$  where  $\Phi$  is the set  $\Phi^A$  of the fields,  $A = 1, \dots, N$ , and with statistics  $\epsilon(\Phi^A) \equiv \epsilon_A$ ,

$$S(\Phi) = F(\Phi) + I(\Phi) , \quad (19)$$

where  $F(\Phi)$  is the kinetic part and  $I(\Phi)$  is the interacting part, which is an analytic function in  $\Phi^A$  around  $\Phi^A = 0$ .

Then one can write conveniently that

$$F(\Phi) = \frac{1}{2} \Phi^A \mathcal{F}_{AB} \Phi^B , \quad (20)$$

and  $\mathcal{F}_{AB}$  is called the kinetic operator.

To perform the NLR we have now to introduce a cut-off or regulating parameter  $\Lambda^2$ . An arbitrary and invertible matrix  $T_{AB}$  has to be introduced too. The combination of  $\mathcal{F}_{AB}$  with  $(T^{-1})^{AB}$  defines a second order derivative regulator:

$$\mathcal{R}_B^A = (T^{-1})^{AC} \mathcal{F}_{CB} . \quad (21)$$

We can construct two important operators with these objects. The first is the smearing operator

$$\epsilon_B^A = \exp \left( \frac{\mathcal{R}_B^A}{2\Lambda^2} \right) , \quad (22)$$

and the second is the shadow kinetic operator

$$\mathcal{O}_{AB}^{-1} = T_{AC} (\tilde{\mathcal{O}}^{-1})_B^C = \left( \frac{\mathcal{F}}{\epsilon^2 - 1} \right)_{AB} , \quad (23)$$

with  $(\tilde{\mathcal{O}})_B^A$  defined as

$$\begin{aligned} \tilde{\mathcal{O}}_B^A &= \left( \frac{\epsilon^2 - 1}{\mathcal{R}} \right)_B^A \\ &= \int_0^1 \frac{dt}{\Lambda^2} \exp \left( t \frac{\mathcal{R}_B^A}{\Lambda^2} \right) . \end{aligned} \quad (24)$$

<sup>1</sup>For convenience we are using the same notation as the reference [23].

In order to expand our original configuration space for each field  $\Phi^A$ , an auxiliary field  $\Psi^A$  can be constructed. We will call these last fields as the shadow fields, with the same statistics as the auxiliary fields. A new auxiliary action involves both sets of fields

$$\tilde{\mathcal{S}}(\Phi, \Psi) = F(\hat{\Phi}) - A(\Psi) + I(\Phi + \Psi) . \quad (25)$$

The second term of this auxiliary action is called the auxiliary kinetic term,

$$A(\Psi) = \frac{1}{2} \Psi^A (\mathcal{O}^{-1})_{AB} \Psi^B . \quad (26)$$

The fields  $\hat{\Phi}^A$ , the smeared fields, which make part of the auxiliary action are defined by

$$\hat{\Phi}^A \equiv (\epsilon^{-1})_B^A \Phi^B . \quad (27)$$

It can be proved that, to eliminate the quantum fluctuations associated with the shadow fields at the path integral level, one has to accomplish this by putting the auxiliary fields  $\Psi$  on shell. So, the classical shadow field equations of motion are

$$\frac{\partial_r \tilde{\mathcal{S}}(\Phi, \Psi)}{\partial \Psi} = 0 \implies \Psi^A = \left( \frac{\partial_r I}{\partial \Phi^B}(\Phi + \Psi) \right) \mathcal{O}^{BA} . \quad (28)$$

These equations can be solved in a perturbative fashion. The classical solutions  $\bar{\Psi}_0(\Phi)$  can now be substituted in the auxiliary action (25). This substitution modify the auxiliary action so that a new action, the non-localized action appear,

$$\mathcal{S}_\Lambda(\Phi) \equiv \tilde{\mathcal{S}}(\Phi, \bar{\Psi}_0(\Phi)) . \quad (29)$$

The action (29) can be expanded in  $\bar{\Psi}_0$ . As a result, we see the appearance of the smeared kinetic term  $F(\hat{\Phi})$ , the original interaction term  $I(\Phi)$  and an infinite series of new non-local interaction terms. But all these interaction terms are  $\mathcal{O}(\Lambda^{-2})$ -like and when the limit  $\Lambda^2 \rightarrow \infty$  is applied, we will have that  $\mathcal{S}_\Lambda(\Phi) \rightarrow \mathcal{S}(\Phi)$ , and the original theory is obtained. Equivalently to this limit, the same result can be acquired with the limits

$$\epsilon \rightarrow 1, \quad \mathcal{O} \rightarrow 0, \quad \bar{\Psi}_0(\Phi) \rightarrow 0 . \quad (30)$$

With all this framework, when we introduce the smearing operator, any local quantum field theory can be made ultraviolet finite. But a question about symmetry can appear. Obviously this form of non-localization, i.e. (27), in general destroy any kind of gauge symmetry or its associated BRST symmetry. The final consequence is the damage of the corresponding Ward identities at the tree level. However, the invariance of the theory can be preserved introducing the auxiliary fields in the original symmetries [23].

Let us make an analysis of what happens. If the original action (19) is invariant under the infinitesimal transformation

$$\delta \Phi^A = R^A(\Phi) , \quad (31)$$

then it can be proved that the auxiliary action is invariant under the auxiliary infinitesimal transformations

$$\begin{aligned} \tilde{\delta} \Phi^A &= (\epsilon^2)_B^A R^B(\Phi + \Psi) , \\ \tilde{\delta} \Psi^A &= (1 - \epsilon^2)_B^A R^B(\Phi + \Psi) . \end{aligned} \quad (32)$$

However, the non-locally regulated action (29) is invariant under the transformation

$$\delta_\Lambda(\Phi^A) = (\epsilon^2)_B^A R^B(\Phi + \bar{\Psi}_0(\Phi)) , \quad (33)$$

remembering that  $\bar{\Psi}_0(\Phi)$  are the solutions of the classical equations of motions (28).

Hence, any of the original continuous symmetries of the theory are preserved at the tree level, even the BRST transformations, and consequently, the original gauge symmetry. The reader can see [1-3] for details.

#### IV. THE EXTENDED (BV) NON-LOCAL REGULARIZATION

As had been said before, the fundamental principle of the field-antifield formalism is the BRST invariance. Therefore, it is simple to realize that the connection of the NLR method with the BV formalism is possible. Using the above construction of the NLR and the BV results, one can build a regulated BRST classical structure of a general gauge theory from the original one. Consequently, a non-locally regularized BV formalism comes out.

We are now in the BV environment. Hence, the configuration space has to be enlarged introducing the antifields  $\{\Psi^A, \Psi_A^*\}$ . Note that the shadow fields have antifields too. Then, an auxiliary proper solution incorporates the auxiliary action (25) (corresponding to the gauge-fixed action  $S(\Phi)$ ), its gauge symmetry (32) and the unknown associated higher order structure functions. The auxiliary BRST transformations are modified by the presence of the term  $\Phi_A^* R^A(\Phi)$  in the original proper solution. Then it can be written that the BRST transformations terms are

$$[\Phi_A^* (\epsilon^2)_B^A + \Psi_A^* (1 - \epsilon^2)_B^A] R^B(\Phi + \Psi) , \quad (34)$$

which are originated from the following substitutions

$$\begin{aligned} R^A &\longrightarrow R^A(\Phi + \Psi) \equiv R^A(\Theta) , \\ \Phi_A^* &\longrightarrow [\Phi_A^* (\epsilon^2)_B^A + \Psi_A^* (1 - \epsilon^2)_B^A] \equiv \Theta_A^* . \end{aligned} \quad (35)$$

For higher orders, the natural way would be

$$R^{A_n \dots A_1}(\Phi) \longrightarrow R^{A_n \dots A_1}(\Phi + \Psi) = R^{A_n \dots A_1}(\Theta) , \quad (36)$$

and an obvious ansatz for the auxiliary proper solution is

$$\begin{aligned} \tilde{S}(\Phi, \Phi^*; \Psi, \Psi^*) &= \tilde{S}(\Phi, \Psi) + \Theta_A^* R^A(\Theta) \\ &+ \Theta_A^* \Theta_B^* R^{BA}(\Theta) + \dots \\ &+ \Theta_{A_1}^* \dots \Theta_{A_n}^* R^{A_n \dots A_1}(\Phi) + \dots \quad (37) \end{aligned}$$

It is intuitive to see that the same canonical conjugation relations, the equations (2), can be obtained, i.e.

$$(\Theta^A, \Theta_B^*) = \delta_B^A . \quad (38)$$

Consequently, we have to construct a new set of fields and antifields  $\{\Sigma^A, \Sigma_A^*\}$  defined by

$$\Sigma^A = \left[ (1 - \epsilon^2)_B^A \Phi^B - (\epsilon^2)_B^A \Psi^B \right] , \quad (39)$$

and

$$\Sigma_A^* = \Phi_A^* - \Psi_A^* . \quad (40)$$

Now we have that the linear transformation

$$\{\Phi^A, \Phi_A^*; \Psi^A, \Psi_A^*\} \longrightarrow \{\Theta^A, \Theta_A^*; \Sigma^A, \Sigma_A^*\} \quad (41)$$

is canonical in the antibracket sense. The auxiliary action (25) is the original proper solution (5) with arguments  $\{\Theta^A, \Theta_A^*\}$ .

The elimination of the auxiliary fields in the non-local BV method is the next step. The shadow fields have to be substituted by the solutions of their classical equations of motion. At the same time, their antifields will be equal to zero. In this way we can write

$$S_\Lambda(\Phi, \Phi^*) = \tilde{S}(\Phi, \Phi^*; \Psi, \Psi^* = 0) , \quad (42)$$

and the classical equations of motion are

$$\frac{\delta_r \tilde{S}(\Phi, \Phi^*; \Psi, \Psi^*)}{\delta \Psi^A} = 0 \quad (43)$$

with solutions  $\bar{\Psi} \equiv \bar{\Psi}(\Phi, \Phi^*)$ , which explicitly read

$$\begin{aligned} \bar{\Psi}^A &= \left[ \frac{\delta_r I}{\delta \Phi^B} (\Phi + \Psi) + \Phi_C^* (\epsilon^2)_D^C R_B^D (\Phi + \Psi) \right. \\ &\quad \left. + O((\Phi^*)^2) \right] \mathcal{O}^{BA} \end{aligned} \quad (44)$$

with

$$R_B^A = \frac{\delta_r R^A(\Phi)}{\delta \Phi^B} . \quad (45)$$

The lowest order of equation (44) is,

$$\bar{\Psi}_0^A = \left( \frac{\delta_r I}{\delta \Phi^B} (\Phi + \Psi) \right) \mathcal{O}^{BA} \quad (46)$$

and one can obtain an expression for  $\bar{\Psi}(\Phi, \Phi^*)$  at any desired order in the antifields [23].

To quantize the theory, it is necessary to add the extra counterterms  $M_p$  to preserve the quantum counterpart of the classical BRST scheme. It is the same as to substitute the classical action  $S$  by a quantum action  $W$ . In the original papers [1–3] the quantization of the theory was already analyzed, but it seems that only the one-loop  $M_1$  corrections acquired BRST invariance. It can be proved that in the field-antifield framework, in general, two-loops and higher order loop corrections should also be considered [23,24].

The complete interaction term,  $\mathcal{I}(\Phi, \Phi^*)$ , of the original proper solution can be written as

$$\mathcal{I}(\Phi, \Phi^*) \equiv I(\Phi) + \Phi_A^* R^A(\Phi) + \Phi_A^* \Phi_B^* R^{BA}(\Phi) + \dots \quad (47)$$

The non-localization of this interaction part furnishes a way to regularize interactions from the counterterms  $M_p$ . To construct the auxiliary free and interactions parts we have that

$$\begin{aligned} \tilde{F}(\Phi + \Psi) &= F(\hat{\Phi}) - A(\Psi) , \\ \mathcal{I}(\Phi, \Phi^*; \Psi, \Psi^*) &= \mathcal{I}(\Theta, \Theta^*) , \end{aligned} \quad (48)$$

with  $\{\Theta, \Theta^*\}$  already known.

Now one have to put the auxiliary fields on shell and its antifields equal to zero, so that

$$\begin{aligned} F_\Lambda(\Phi, \Phi^*) &= \tilde{F}(\Phi, \bar{\Psi}_0) , \\ \mathcal{I}_\Lambda(\Phi, \Phi^*) &= \tilde{\mathcal{I}}(\Phi + \bar{\Psi}_0, \Phi^* \epsilon^2) , \end{aligned} \quad (49)$$

then  $S_\Lambda = F_\Lambda + \mathcal{I}_\Lambda$ .

The quantum action  $W$  can be expressed by

$$W = F + \mathcal{I} + \sum_{p=1}^{\infty} \hbar^p M_p \equiv F + \mathcal{Y} \quad (50)$$

where  $\mathcal{Y}$  is the generalized quantum interaction part.

An analogous procedure of the previous section can be applied to the quantum action  $W$ . We will omit all the formal steps here. All the details can be founded in ref. [23,24].

A decomposition in its divergent part and its finite part when  $\Lambda^2 \longrightarrow \infty$  can be accomplished in the regulated QME.

It can be shown that the expression of the anomaly is the value of the finite part in the limit  $\Lambda^2 \longrightarrow \infty$  of

$$\mathcal{A} = \left[ (\Delta W)_R + \frac{i}{2\hbar} (W, W) \right] (\Phi, \Phi^*) \quad (51)$$

and the regularized value of  $\Delta W$  is defined as

$$(\Delta W)_R \equiv \lim_{\Lambda^2 \rightarrow \infty} [\Omega_0] \quad (52)$$

where

$$\Omega_0 = \left[ S_B^A (\delta_\Lambda)_C^B (\epsilon^2)_A^C \right]. \quad (53)$$

$(\delta_\Lambda)_B^A$  is defined by

$$\begin{aligned} (\delta_\Lambda)_B^A &= (\delta_B^A - \mathcal{O}^{AC} \mathcal{I}_{CB})^{-1} \\ &= \delta_B^A + \sum_{n=1} (\mathcal{O}^{AC} \mathcal{I}_{CB})^n, \end{aligned} \quad (54)$$

with

$$\begin{aligned} S_B^A &= \frac{\delta_r \delta_l S}{\delta \Phi^B \delta \Phi_A^*}, \\ \mathcal{I}_{AB} &= \frac{\delta_r \delta_l \mathcal{I}}{\delta \Phi^A \delta \Phi^B}. \end{aligned} \quad (55)$$

Applying the limit  $\Lambda^2 \rightarrow \infty$  in (52), it can be shown that

$$(\Delta S)_R \equiv \lim_{\Lambda^2 \rightarrow \infty} [\Omega_0]_0, \quad (56)$$

and finally that

$$\begin{aligned} \mathcal{A}_0 &\equiv (\Delta S)_R \\ &= \lim_{\Lambda^2 \rightarrow \infty} [\Omega_0]_0. \end{aligned} \quad (57)$$

All the higher orders terms of the anomaly can be obtained from equation (51), but this will not be analyzed in this paper. It can be seen in [24].

## V. THE EXTENDED NON-LOCAL REGULARIZATION OF THE CHIRAL SCHWINGER MODEL

In this section we will make a comparison between the results of the computation of the anomaly of the CSM using two different regulators.

### A. Second order differential regulator

The classical action for the chiral Schwinger model is

$$\begin{aligned} S &= \int d^2x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{\partial} \psi \right. \\ &\quad \left. + \frac{e}{2} \bar{\psi} \gamma_\mu (1 - \gamma_5) A^\mu \psi \right], \end{aligned} \quad (58)$$

which obviously has a perturbative expansion<sup>2</sup>.

This action is invariant under the following gauge transformations:

$$A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \theta(x) \quad (59)$$

$$\psi(x) \longrightarrow \exp [i e (1 - \gamma_5) \theta(x)] \psi(x). \quad (60)$$

The kinetic part of the action (58) is given by

$$\begin{aligned} F &= \int d^2x \bar{\psi} i \not{\partial} \psi \\ &= \int d^2x \left[ \frac{1}{2} \bar{\psi} i \not{\partial} \psi + \frac{1}{2} \bar{\psi} i \not{\partial} \psi \right]. \end{aligned} \quad (61)$$

Integrating by parts the second term we have that

$$F = \int d^2x \left[ \frac{1}{2} \bar{\psi} i \not{\partial} \psi + \frac{1}{2} \psi (i \not{\partial}^t \bar{\psi}) \right]. \quad (62)$$

The kinetic term has the form

$$F = \frac{1}{2} \Psi^A \mathcal{F}_{AB} \Psi^B. \quad (63)$$

So,

$$\Psi = \begin{pmatrix} \bar{\psi} \\ \psi \end{pmatrix} \quad (64)$$

and

$$F = \frac{1}{2} (\bar{\psi} \ \psi) \begin{pmatrix} 0 & i \not{\partial} \\ i \not{\partial}^t & 0 \end{pmatrix} \begin{pmatrix} \bar{\psi} \\ \psi \end{pmatrix}. \quad (65)$$

The kinetic operator ( $\mathcal{F}_{AB}$ ) is defined by

$$\mathcal{F}_{AB} = \begin{pmatrix} 0 & i \not{\partial} \\ i \not{\partial}^t & 0 \end{pmatrix}. \quad (66)$$

The regulator, a second order differential operator, is

$$\mathcal{R}_\beta^\alpha = (T^{-1})^{\alpha\gamma} \mathcal{F}_{\gamma\beta}, \quad (67)$$

where  $T$  is an arbitrary matrix, hence one can make the following choice:

$$\mathcal{R}_\beta^\alpha = -\partial^2. \quad (68)$$

Let us define the smearing operator,

$$\epsilon_B^A = \exp \left( \frac{-\partial^2}{2\Lambda^2} \right), \quad (69)$$

and the smeared fields

$$\hat{\Phi}^A = (\epsilon^{-1})_B^A \Phi^B. \quad (70)$$

In the NLR scheme the shadow kinetic operator is

$$\mathcal{O}_{\alpha\beta}^{-1} = \left( \frac{\mathcal{F}}{\epsilon^2 - 1} \right)_{\alpha\beta} \quad (71)$$

then

$$\mathcal{O} = \begin{pmatrix} 0 & -i \mathcal{O}' \not{\partial} \\ -i \mathcal{O}' \not{\partial}^t & 0 \end{pmatrix} \quad (72)$$

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<sup>2</sup>We are using the Einstein notation.

where  $\mathcal{O}'$  is defined by

$$\begin{aligned}\mathcal{O}' &= \frac{\epsilon^2 - 1}{\partial^t \partial} \\ &= \int_0^1 \frac{dt}{\Lambda^2} \exp\left(t \frac{\partial^t \partial}{\Lambda^2}\right) ,\end{aligned}\quad (73)$$

notice that we have to obey the rules for the product of the Dirac matrices  $\gamma_\mu$ .

The interacting part of the action (58) is

$$I[A_\mu, \psi, \bar{\psi}] = \frac{e}{2} \bar{\psi} \gamma_\mu (1 - \gamma_5) A^\mu \psi , \quad (74)$$

$$\begin{aligned}I[A_\mu, \psi + \Phi, \bar{\psi} + \bar{\Phi}] &= \frac{e}{2} (\bar{\psi} + \bar{\Phi}) \gamma_\mu (1 - \gamma_5) \\ &\quad \times A^\mu (\psi + \Phi) ,\end{aligned}\quad (75)$$

where  $\Phi$  are the shadow fields.

The BRST transformations are given by

$$\begin{aligned}\delta A_\mu &= \partial_\mu c , \\ \delta \psi &= i(1 - \gamma_5) \psi c , \\ \delta \bar{\psi} &= -i\bar{\psi}(1 + \gamma_5) c , \\ \delta c &= 0 .\end{aligned}\quad (76)$$

Using the equations (35), where the antifields are functions of the auxiliary fields,

$$\begin{aligned}\psi^* &\longrightarrow [\psi^* \epsilon^2 + \Phi^* (1 - \epsilon^2)] , \\ \bar{\psi}^* &\longrightarrow [\bar{\psi}^* \epsilon^2 + \bar{\Phi}^* (1 - \epsilon^2)] .\end{aligned}\quad (77)$$

The generator of the BRST transformations are

$$\begin{aligned}R(\psi) &\longrightarrow R(\psi + \Phi) = i(1 - \gamma_5) (\psi + \Phi) c , \\ R(\bar{\psi}) &\longrightarrow -i(\bar{\psi} + \bar{\Phi}) (1 + \gamma_5) c , \\ R(c) &= 0 .\end{aligned}\quad (78)$$

We are able now to construct the non-local auxiliary proper action. It will be given in general by

$$S_\Lambda(\Phi, \Phi^*) = \tilde{S}_\Lambda(\Phi, \Phi^*; \psi_s, \psi^* = 0) , \quad (79)$$

where  $\psi_s$  are the solutions of the classical equations of motion.

The proper solution, the BV action, is given by

$$\begin{aligned}S_{BV} &= \int d^2x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{\partial} \psi \right. \\ &\quad + \frac{e}{2} \bar{\psi} \gamma_\mu (1 - \gamma_5) A^\mu \psi + A_\mu^* \partial^\mu c \\ &\quad \left. + i \psi^* (1 - \gamma_5) \psi c - i \bar{\psi}^* \bar{\psi} (1 + \gamma_5) c \right] .\end{aligned}\quad (80)$$

After a tedious algebra, one can write the non-localized action as

$$\begin{aligned}\tilde{S}_\Lambda(\psi, \psi^*) &= -\frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} + \hat{\psi} i \not{\partial} \hat{\psi} + A_\mu^* \partial^\mu c \\ &\quad + \frac{e}{2} \frac{(i \not{\partial}) [\bar{\psi} \gamma^\mu (1 - \gamma_5) A_\mu \psi]}{i \not{\partial} + e \gamma^\mu (1 - \gamma_5) A_\mu (\epsilon^2 - 1)} \\ &\quad + \frac{i \psi^* \epsilon^2 c (-i \not{\partial}) (1 - \gamma_5) \psi}{[i \not{\partial} + e \gamma^\mu (1 - \gamma_5) A_\mu (\epsilon^2 - 1)]} \\ &\quad + \frac{i \bar{\psi}^* \epsilon^2 c (i \not{\partial}) \bar{\psi} (1 + \gamma_5)}{[i \not{\partial} + e \gamma^\mu (1 - \gamma_5) A_\mu (\epsilon^2 - 1)]} .\end{aligned}\quad (81)$$

It can be easily seen that when one take the limit  $\epsilon^2 \longrightarrow 1$ , the original proper solution  $S_{BV}$  of the CSM is obtained. This is a representative expression, since it is well known that operators in the denominator of any expression are physically senseless.

The final part is the computation of the one-loop anomaly of the chiral Schwinger model. Firstly, we have to construct some very important matrices,

$$S_B^A = \frac{\delta_r \delta_l S_{BV}}{\delta \Phi^B \delta \Phi_A^*} \quad (82)$$

Then

$$S_B^A = \begin{pmatrix} -ic(1 - \gamma_5) & 0 \\ 0 & ic(1 + \gamma_5) \end{pmatrix} . \quad (83)$$

The operator  $\mathcal{I}_{AB}$  in this case is defined by,

$$\mathcal{I}_{AB} = \frac{\delta_l \delta_r [I(\Phi) + \Phi_c^* R^c(\Phi)]}{\delta \Phi^A \delta \Phi^B} \quad (84)$$

and the result is,

$$\mathcal{I}_{AB} = \begin{pmatrix} 0 & -\frac{e}{2} \gamma_\mu (1 - \gamma_5) A^\mu \\ \frac{e}{2} \gamma_\mu (1 - \gamma_5) A^\mu & 0 \end{pmatrix} . \quad (85)$$

The one-loop anomaly is given by:

$$\mathcal{A} \equiv (\Delta S)_R , \quad (86)$$

$$(\Delta S)_R = \lim_{\Lambda^2 \rightarrow \infty} [\Omega_0]_0 , \quad (87)$$

$$\begin{aligned}\Omega_0 &= [\epsilon^2 S_A^A] + [\epsilon^2 S_B^A \mathcal{O}^{BC} \mathcal{I}_{CA}] \\ &\quad + O\left(\frac{(\Phi^*)^2}{\Lambda^2}\right) .\end{aligned}\quad (88)$$

For the first term we can compute that

$$\begin{aligned}\epsilon^2 S_A^A &= \epsilon^2 \text{tr} S_B^A \\ &= 0 ,\end{aligned}\quad (89)$$

now we have that

$$(\Delta S)_R = \lim_{\Lambda^2 \rightarrow \infty} \text{tr} [\epsilon^2 S_B^A \mathcal{O}^{BC} \mathcal{I}_{CA}] . \quad (90)$$

Using the Weyl representation of the  $\gamma$  matrices in two dimensions in Euclidian space:

$$\begin{aligned}
\gamma_0 &= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} , \\
\gamma_1 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \\
\gamma^5 &= -i \gamma_1 \gamma_0 , 
\end{aligned} \tag{91}$$

and that in this representation,  $\gamma_5^t = \gamma_5$  .

Finally, after some algebra

$$(\Delta S)_R = \lim_{\Lambda^2 \rightarrow \infty} \text{tr} \left[ \epsilon^2 (-ec) \mathcal{O}' (\gamma^\nu)^t \gamma^\mu \right. \\
\left. \times (1 - \gamma_5) \partial_\nu A_\mu \right] \tag{92}$$

$$= \lim_{\Lambda^2 \rightarrow \infty} \text{tr} \left[ \epsilon^2 (-ec) \frac{\epsilon^2 - 1}{\partial^2} \right. \\
\left. \times (\partial_\mu A^\mu - i \epsilon^{\mu\nu} \partial_\mu A_\nu) \right] . \tag{93}$$

But we know that the functional traces can be written as

$$\lim_{\Lambda^2 \rightarrow \infty} \text{tr} \left[ \epsilon^2 F \partial^n \frac{\epsilon^2 - 1}{\partial^2} \partial G \partial^m \right] = \tag{94}$$

$$= \frac{-i}{2\pi} \left[ \sum_{k=0}^m \binom{m}{k} \frac{(-1)^k}{n+m+1-k} \left( 1 - \frac{1}{2^{n+m+1-k}} \right) \right] \\
\times \int d^2 x F \partial^{n+m+1} G , \tag{95}$$

this last equation can be derived from (73) after hard work where convenient reparametrization were necessary.

In our case

$$\begin{aligned}
n &= m = 0 \\
F &= -ec \\
\partial G &= \partial_\mu A^\mu - i \epsilon^{\mu\nu} \partial_\mu A_\nu , 
\end{aligned} \tag{96}$$

and the final result is

$$\mathcal{A} = (\Delta S)_R = \frac{ie}{4\pi} \int d^2 x c (\partial_\mu A^\mu - i \epsilon^{\mu\nu} \partial_\mu A_\nu) \tag{97}$$

which is exactly the one-loop anomaly of the chiral Schwinger model, action (58).

## B. The Fujikawa regulator (FR)

We will study now the utilization of a second regulator, a FR to get the same result as was obtained above. Originally, the FR is a covariant derivative that was used by Fujikawa to calculate the chiral anomaly [25]. Only the main steps of the calculation are presented here.

Notice that now we know the value of the anomaly. We will show here that there is another different regulator that furnish the same result. To do this let us construct a convenient FR given by

$$\mathcal{R}_\beta^\alpha = \mathcal{D} = \gamma_\mu (\partial^\mu + A^\mu) , \tag{98}$$

introducing this regulator à la Fujikawa in (92) we can write that

$$\begin{aligned}
(\Delta S)_R &= \\
&= \lim_{\Lambda^2 \rightarrow \infty} \text{tr} \left\{ \epsilon^2 (-ec) \int \frac{d^2 k}{(2\pi)^2} e^{-ikx} \right. \\
&\times \int_0^1 \frac{dt}{\Lambda^2} \exp \left( t \frac{\mathcal{D} \mathcal{D}^t}{\Lambda^2} \right) \\
&\times (\gamma^\nu)^t \gamma^\mu (1 - \gamma_5) \partial_\nu A_\mu \exp \left( -\frac{\mathcal{D}^2}{\Lambda^2} \right) e^{ikx} \left. \right\} \\
&= \lim_{\Lambda^2 \rightarrow \infty} \text{tr} \left\{ \epsilon^2 (-ec) \int_0^1 \frac{dt}{\Lambda^2} \exp \left( t \frac{\mathcal{D} \mathcal{D}^t}{\Lambda^2} \right) \right. \\
&\times (\gamma^\nu)^t \gamma^\mu (1 - \gamma_5) \partial_\nu A_\mu \\
&\times \exp \left( -\frac{1}{\Lambda^2} \left[ A^2 + \partial^\mu A_\mu + \frac{1}{4} [\gamma^\mu, \gamma^\nu] F_{\mu\nu} \right] \right) \\
&\times \int \frac{d^2 k}{(2\pi)^2} \exp \left[ -\frac{1}{\Lambda^2} (k^2 + 2i A_\mu k^\mu) \right] \left. \right\} . \tag{99}
\end{aligned}$$

Notice that we have substituted the  $\mathcal{O}'$  operator by its integral form.

It is necessary to make a usefull transformation of the  $k^\mu$  coordinate,

$$k^\mu \longrightarrow \Lambda k^\mu . \tag{100}$$

Expanding the exponential of  $t$  it is easy to realize that only the unitary term of the expansion can be used, since the other terms have the  $\Lambda^{-n}$  form, and hence disappear with the infinity limit. After the computation of the integrals, we have that,

$$(\Delta S)_R = \frac{ie}{4\pi} \text{tr} \left\{ (\gamma^\nu)^t \gamma^\mu (1 - \gamma_5) \partial_\nu A_\mu \right\} . \tag{101}$$

Manipulating with the  $\gamma$  matrices (Weyl representation), the final result is

$$\mathcal{A} = (\Delta S)_R = \frac{ie}{4\pi} \int d^2 x c (\partial_\mu A^\mu - i \epsilon^{\mu\nu} \partial_\mu A_\nu) , \tag{102}$$

which is the same result as we have obtained before with a different regulator.

As has been said before, it is a very interesting result because it was expected that the expressions for the anomaly would be not equal, as has been showed in [27] and [28].

## VI. CONCLUSIONS

The non-local regularization formalism is a recent and a quite powerfull method to regularize theories with a perturbative expansion which have higher order loop divergences. The field-antifield framework exhibits a divergence in the application of the  $\Delta$  operator. Hence it

needs a regularization. The connection between the BV formalism and the NLR method generates an extended non-locally regularized BV quantization method. At the quantum level its use in the path integral originates this extended BV formalism through the construction of a non-local regularized quantum action. In this way we can compute higher order loop of the BRST anomaly that is contained in the quantum master equation. To make this connection, we have to introduce auxiliary fields with which we have constructed an auxiliary proper solution of the master equation. These auxiliary fields are eliminated from the theory through the field equations. At this point, and with some technical work, we can construct a regularized  $\Delta S$  expression that furnishes the final form of the anomaly.

In this work, the theory used was a fermionic one, the chiral Schwinger model. The objective is to analyze the results of the anomaly using two different regulators: a second order differential one and a kind of covariant derivative operator, like the one that was used by Fujikawa to calculate the chiral anomaly.

With the second regulator, at a certain point of the process, the non-local characteristic of the method, contained in the  $\mathcal{O}'$  operator, was substituted by its integral form and furthermore it has been combined with the Fujikawa formalism to give an exact result.

We know that some anomalous theories can have the anomaly calculation dependent on the regulator that has been used. It was shown in the literature that the solutions for the CSM have different dependences on the parameters of the regulators. However, these parameters are free to be chosen, causing no conflict between the results. The final form of the anomaly can not be obtained in a direct way, and consequently, we have to make a reparametrization to obtain the final answer. In another way it was also demonstrated that we have to introduce a local counterterm, the WZ term, to obtain the equivalence between both different expressions obtained for the anomaly. In this work, we have shown in a precise way that the calculation of the one-loop anomaly of the CSM with different regulators has furnished directly the same final results. Finally, we can observe that it would be very interesting to make the same analysis for theories in higher dimensions.

**Acknowledgment:** the author would like to thank Nelson R. F. Braga and Clovis Wotzasek for valuable discussions and suggestions. This work is supported by CAPES (Brazilian Research Agency).

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